FORMULÁRIO - 17/05/2007

ERRO QUADRÁTICO

$$EQ(f,g) = \|f-h\| = \sqrt{\langle\ f-g \mid f-g\ \rangle}$$

POLINÔMIOS ORTOGONAIS MÔNICOS - FÓRMULAS DE RECORRÊNCIA

$$p_{-1}(x)=0,\quad p_0(x)=1,\quad p_k(x)=(x-\alpha_k)p_{k-1}(x)-\beta_k p_{k-2}(x),\quad k=1,2,\ldots$$

$$\text{onde}\quad \alpha_k = \frac{\langle\ xp_{k-1} \mid p_{k-1}\ \rangle}{\langle\ p_{k-1} \mid p_{k-1}\ \rangle}\quad \beta_k = \frac{\langle\ xp_{k-1} \mid p_{k-2}\ \rangle}{\langle\ p_{k-2} \mid p_{k-2}\ \rangle}\quad (\beta_1 = 0)$$

ANÁLISE HARMÔNICA PARA f DE PERÍODO $2L$

$$g(x) = a_0 + \sum_{k=1}^{m}\left[a_k\cos\left(k\frac{\pi}{L}x\right) + b_k\sin\left(k\frac{\pi}{L}x\right)\right]\quad \text{onde}$$

Caso Contínuo: f integrável em $[a, a+2L]$	Caso Discreto: f tabelada em $x_j = a + j\frac{\pi}{L},\ j=1,\ldots,2N$
$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx = \frac{1}{2L}\int_{a}^{a+2L} f(x)dx$ $a_k = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(k\frac{\pi}{L}x\right)dx = \frac{1}{L}\int_{a}^{a+2L} f(x)\cos\left(k\frac{\pi}{L}x\right)dx$ $b_k = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(k\frac{\pi}{L}x\right)dx = \frac{1}{L}\int_{a}^{a+2L} f(x)\sin\left(k\frac{\pi}{L}x\right)dx$ $k=1,2,\ldots,m$	$a_0 = \frac{1}{2N}\sum_{j=1}^{2N} f(x_j)$ $a_k = \frac{1}{N}\sum_{j=1}^{N} f(x_j)\cos(kx_j)$ $b_k = \frac{1}{N}\sum_{j=1}^{N} f(x_j)\sin(kx_j)$ $k=1,2,\ldots,N$

POLINÔMIOS DE LEGENDRE

$$p_0(x)=1,\quad p_1(x)=x,\quad p_2(x)=\frac{1}{2}(3x^2-1),\quad p_3(x)=\frac{1}{2}(5x^3-3x),\ldots$$

$$\text{satisfazem}\quad \int_{-1}^{1} p_i(x)p_j(x)dx = \begin{cases} 0, & \text{se } i \neq j \\ \frac{2}{2i+1}, & \text{se } i = j \end{cases}$$

INTERPOLAÇÃO POLINOMIAL DE F RELATIVO AOS PONTOS DISTINTOS $x_0, x_1, \ldots, x_n$

(a) Forma de Lagrange:

$$p_n(x) = \sum_{k=0}^{n} f(x_k)L_k(x),\quad \text{onde}\quad L_k(x) = \prod_{\substack{0\le r\le n\\ r\neq k}} \frac{(x-x_r)}{(x_k-x_r)}$$

(b) Diferenças Divididas:

$$f[x_i] = f(x_i), i=0,1,\ldots,n$$

$$f[x_i, x_{i+1},\ldots,x_{j-1},x_j] = \frac{f[x_{i+1},\ldots,x_{j-1},x_j] - f[x_i,x_{i+1},\ldots,x_{j-1}]}{x_j - x_i},\ 0\le i<j\le n$$

(c) Diferenças Simples:

$$
\begin{aligned}
h > 0 \quad \Delta^0 f(x) &= \quad f(x) \\
\Delta^1 f(x) &= \quad \Delta f(x) = f(x+h) - f(x) \\
\Delta^n f(x) &= \quad \Delta(\Delta^{n-1} f)(x) = \Delta^{n-1} f(x+h) - \Delta^{n-1} f(x), \quad n \geq 2
\end{aligned}
$$

(d) Relação: Se $x_j = x_0 + h,\ j = 0, 1, \ldots, n$ então

$$f[x_0, x_1, \ldots, x_k] = \frac{\Delta^k f(x_0)}{k! h^k}, \quad k = 0, 1, \ldots, n$$

(e) Forma de Newton:

$$
\begin{aligned}
p_n(x) \quad &= \quad f[x_0] + f[x_0, x_1](x - x_0) + \cdots + f[x_0, x_1, \ldots, x_n](x - x_0)(x - x_1) \cdots (x - x_{n-1}) \\
& \quad\ \text{ou} \\
p_n(x) \quad &= \quad \frac{\Delta^0 f(x_0)}{0! h^0} + \frac{\Delta^1 f(x_0)}{1! h^1}(x - x_0) + \cdots + \frac{\Delta^n f(x_0)}{n! h^n}(x - x_0)(x - x_1) \cdots (x - x_{n-1})
\end{aligned}
$$

(f) Erro de truncamento no ponto $x \in [a, b]$ quando f é de classe C^{n+1} e $x_0, x_1, \ldots, x_n \in [a, b]$:

$$E(x) = f(x) - p_n(x) = \frac{(x - x_0)(x - x_1) \cdots (x - x_{n-1})(x - x_n)}{(n+1)!} f^{n+1}(\xi_x),$$

para algum $\xi_x \in [a, b]$.

INTEGRAÇÃO NUMÉRICA

(a) Método dos Trapézios: $[a, b] = [x_0, x_n]$

$$\int_a^b f(x)dx \cong \frac{h}{2}\left[f(x_0) + f(x_n) + 2\sum_{k=1}^{n-1} f(x_k)\right]$$

Erro de truncamento se f é de classe C^2 em $[a, b]$:

$$|E_T| \leq \frac{(b-a)^3}{12n^2} \max_{z \in [a,b]} |f''(z)|$$

(b) Método de Simpson: $[a, b] = [x_0, x_{2n}]$

$$\int_a^b f(x)dx \cong \frac{h}{3}\left[f(x_0) + f(x_{2n}) + 4\sum_{k=1}^{n} f(x_{2k-1}) + 2\sum_{k=1}^{n-1} f(x_{2k})\right]$$

Erro de truncamento se f é de classe C^4 em $[a, b]$:

$$|E_S| \leq \frac{(b-a)^5}{2880n^4} \max_{z \in [a,b]} |f^{(4)}(z)|$$